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Why are Black Holes Stable Against Their Own Gravity?

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One of the great mysteries of physics is why neutron stars, which are timelike matter, have a maximum mass, but Black Holes, which are spacelike matter, have no maximum mass. Indeed, Black Holes have been observed having masses of billions of solar masses. The answer ultimately is related to the stability of Black Holes (BH) against their own gravity, which is the key to understanding General Relativity. We show that the answer to these two related questions is the existence of a fundamental Black Hole constant: $\mathcal{F} = \frac{3c^4}{4G} = 9.077\ldots \times 10^{43}$ Newtons, independent of Black Hole mass. This constant allows us to derive the correct area law for Black Hole coalesce.

Keywords: Neutron star; scalar curvature; black hole.

1. Introduction

The goal in this paper is to explain why Black Holes are stable against their own gravity and, in so doing, reach an understanding why BH have no maximum mass.[a] In discussing Black Hole physics, the salient observation is that the Black Hole interior is spacelike,[1–4] meaning that the particles are off their mass shell in the spacelike region. Spacelike matter is *acausal* and only gravitational invariants (observables at infinity and R, the scalar curvature) are present. This means for BH:

(1) There are no identifiable particle states.
(2) There is no Pauli Principle.
(3) There is no equation of static equilibrium.
(4) There is no microscopic equation of state.

[a] There may be practical limits on the efficiency of accretion disks for super-massive Black Holes.

(5) There are no equations of motion. In particular, the Einstein Field equations, which are causal, do not apply.
(6) Spacelike matter has no entropy.
(7) Spacelike matter has no temperature.
(8) There are no identifiable particle interactions. For example, no weak nuclear force.
(9) There is no Planck constant.
(10) There is no Boltzmann constant.
(11) Applying finite temperature field theory, which is causal, leads to contradictions (discussed below).

Since the Einstein field equations do not apply inside a BH, what replaces them? The answer is that a gravitational invariant constituent equation does, and we discuss its derivation below. The important consideration is that in timelike matter, the renormalization of R must satisfy the same theorem[5] as the renormalization of the thermodynamic potential, Ω, in finite temperature field theory, because both are observable with no Green's Function external legs (fields). We show that if you insist on applying causal physics to acausal BH, you get contradictions.

The correct area theorem for coalescing BH falls out. It is highly improbable that three or more BH coalesce, but we give the formula for N-number. BH have an intrinsic negative scalar curvature[6] which gives an outward pressure. The outward pressure exactly cancels the inward self-gravity pressure and so the total pressure inside a BH is exactly zero (and so there is no surface tension). Furthermore we show that this equilibrium is stable and BH have no singularities.

2. Stability of Neutron Stars

To understand the stability of BH against gravity, it is advantageous to look at the stability of a neutron star (NS). A non-rotating NS is the solution of the Oppenheimer–Volkoff (O–V) General Relativistic equation of hydrostatic equilibrium, with an equation of state (EOS). The EOS (mass density versus pressure) starts from the NS surface crystalline crust down to its core. Using Quantum Chromodynamics for the core region in finite temperature field theory, Kislinger and Morley[7] derived the maximum mass of a NS to be 2.34 $M_{\odot}$, with a central density $10^{15}\,\mathrm{gm/cm^3}$. In general, the O–V NS solutions look like Fig. 1, where the NS asymptotic mass is depicted as a function of its radius.

Points A, B and C highlight the three regions, which are, respectively, negative slope, zero slope and positive slope. We start from radii in the negative slope region (large radius) and go inwards; our interest is in radial stability of the NS. At point A, we make a radial perturbation to a smaller radius. The star has a greater mass for the smaller perturbed radius, so the NS has radial stability. The whole negative slope region is the physical region of NS that exist in the Universe. This negative slope region is critical:

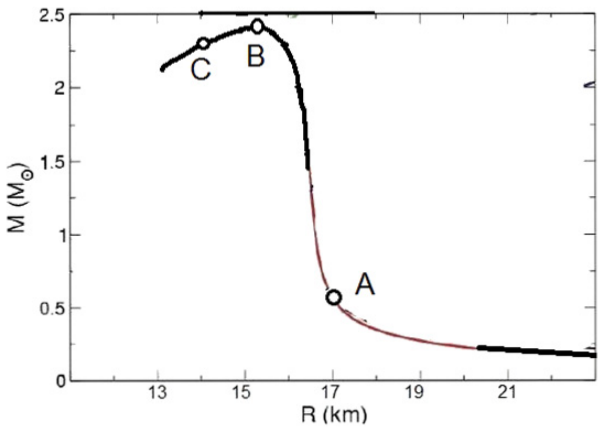

Fig. 1. Neutron Star mass versus radius O–V solutions. Points A, B and C are discussed in the text.

(1) It is the signature for degenerate matter.
(2) The signature is: smaller radii have higher mass.
(3) True for NS and white dwarfs (electron degeneracy).
(4) True for the condensation of the cosmological neutrinos, whose structures are so large, their radii are millions of light years in size and embed whole galaxy clusters.

Next, we examine point B. This is the maximum NS mass that can exist, for non-rotating NS. Beyond the central density (smaller radius) associated with point B, we have radial instability: If we start at point C and do a radial perturbation to a smaller radius, the star has a smaller mass (due to gravitational self-interaction) and the NS rolls down the positive slope, gaining infall energy from the smaller masses. The positive branch for NS therefore does not exist. Our assumed lesson from NS:

(1) Positive slopes for mass versus radius are radially unstable.
(2) Timelike degenerate matter has a maximum mass.

3. Joint Matter-Gravity Action Functional

The General Relativistic solution for non-rotating (static) BH is the Schwarzschild solution, which has mass M versus radius r_0 (where G, c are, respectively, the gravitational constant and speed of light and r_0 is the Schwarzschild radius).

$$M = \frac{c^2}{2G} r_0. \tag{1}$$

Equation (1) is a straight-line, positive slope relationship and would indicate that any BH should radially collapse, gaining infall energy from the increasingly more bound gravitational self-mass. So it would seem no stable BH can exist, only an eventual singularity. In addition to this apparent instability, we already mentioned that BH have no upper bound to their mass.

In order to make progress, we have to go back to 1915 when Albert Einstein was creating General Relativity. He had two equally important tasks that were independent of each other:

(1) What is the correct gravitational Lagrangian.
(2) What is the correct joint matter-gravity action.

All experiments to date prove that Einstein got 1 correct. For 2, Einstein guessed that the joint gravity-matter action is the Einstein–Hilbert formula

$$A_{\mathrm{EH}} = \int \left\{ \frac{R}{2\kappa} + L_{\mathrm{SM}} \right\} \sqrt{-g}\; dx^3 dt, \tag{2}$$

where we use the $(-+++)$ metric signature, the constant $\kappa = \frac{8\pi G}{c^4}$, where R is the scalar curvature, $g = \det[g_{\mu\nu}]$ and L_{SM} is the Standard Model Lagrangian density. Though gravity was known to be nonlinear, Eq. (2) is linear in the two Lagrangian densities. This linearity has consequences[6]:

(1) Gravity perturbations using Eq. (2) are not renormalizable in four-dimensional spacetime.[8,9]
(2) The action does not vanish when $L_{\mathrm{SM}} \to 0^-$, so spacetime exists without any energy/mass in the Universe. The reason why 0^- is required is that $L_{\mathrm{SM}} \to -$ (energy density).
(3) The action is the complex phase of the probability amplitude and the addition of the gravitational phase $\int \frac{R}{2\kappa} \sqrt{-g}\, dx^3 dt$ with the energy/mass phase $\int L_{\mathrm{SM}} \sqrt{-g}\, dx^3 dt$ means that the *joint probability* for pure states in the Hilbert Space defined by Eq. (2) is just the product of the gravitational probability with the energy/mass probability, i.e., *energy/mass is not correlated with gravity for the pure states.*

The only way to ensure that gravity, spacetime and matter/energy are always correctly correlated to each other is to have a nonlinear A_{joint} action functional that vanishes when $L_{\mathrm{SM}} \to 0^-$.

A joint matter-gravity action that removes these deficiencies is the following (see[10])

$$A_{\mathrm{joint}} = \int e^{\frac{R}{2QL_{\mathrm{SM}}}} L_{\mathrm{SM}} \sqrt{-g} dr^3 dt, \tag{3}$$

where $L_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times U(1)_Y$, the Standard Model and Q is a constant to be determined. Taking the extremum[10] gives (with $Q = 12\frac{\pi G}{c^4}$)

$$R^{\mathrm{field}}_{\mu\nu} - \frac{R^{\mathrm{field}}}{2} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{4}$$

where $T_{\mu\nu}$ is the stress-energy tensor, and a new additional equation

$$R^{\text{intrinsic}} = \frac{8\pi G}{c^4} L_{\text{SM}}. \tag{5}$$

We have the usual Einstein equations of motion (4) and a new equation (5). This new equation is missing from General Relativity. One can consider it the *gravitational constituent equation*, because it relates two disparate physical quantities, R and L_{SM}. We explain now the underlying mathematics of this equation.

The scalar curvature R is invariant under isometric symmetries (distance invariance) associated with metric spaces. Gravitational Riemannian manifolds are metric spaces. The Standard Model L_{SM} has the same isometric symmetry as R and the joint action predicts that $R^{\text{intrinsic}}$ and L_{SM} are proportional to each other, with the constant of proportionality $\frac{8\pi G}{c^4}$. So in any spacetime manifold, L_{SM} has its usual internal symmetries (gauge invariance, etc.) and has the isometric symmetry of the spacetime it is embedded in. For Minkowsi Space, the isometric symmetry is the Poincaré group.

The difference between R^{field} and $R^{\text{intrinsic}}$ was brought out in Ref. 6. Here, we wish to give a more detailed understanding of these two scalar curvatures. $R^{\text{intrinsic}}$ contributes to the (classical) Einstein field equations by the constant $\frac{12\pi G}{c^4}$ in the nonlinear joint action: Eq. (3) $\rightarrow \frac{8\pi G}{c^4}$ in the field equations (4). $R^{\text{intrinsic}}$ presence as a separate equation in Eq. (5) means that it becomes an operator equation in quantum theory allowing the renormalization of R[10] for timelike matter:

$$\begin{aligned} R_R(\{g_{Ri}\},\{m_{RI}\},T,\{\mu_i\}) &= R_B(\{g_{Bi}\},\{m_{BI}\},T,\{\mu_i\}) \\ &\quad - R_B(\{g_{Bi}\},\{m_{BI}\},T=0,\{\mu_i=0\}), \end{aligned} \tag{6}$$

where B and R refer to bare and renormalized quantities, $\{g_i\}$, $\{m_i\}$ refer to the collection of coupling constants and masses and T, $\{\mu_i\}$ refer to temperature and the various chemical potentials.

This is the same renormalization as the thermodynamic potential Ω in finite temperature field theory

$$\begin{aligned} \Omega_R(\{g_{Ri}\},\{m_{RI}\},T,\{\mu_i\}) &= \Omega_B(\{g_{Bi}\},\{m_{BI}\},T,\{\mu_i\}) \\ &\quad - \Omega_B(\{g_{Bi}\},\{m_{BI}\},T=0,\{\mu_i=0\}). \end{aligned} \tag{7}$$

In finite temperature field theory, the thermodynamic potential Ω and the gravitational scalar R have the same renormalization theorem for timelike matter: Both are physical observables without any Green's function external lines (fields). It should be pointed out that the infamous 10^{120} difference between the electroweak vacuum energy and the experimental vacuum energy cancels in the right-hand side of Eq. (7).

Table 1 gives a summary of the two scalar curvatures $R^{\text{intrinsic}}$ and R^{field}.

Table 1. Timelike and spacelike contributions of $R^{\text{intrinsic}}$.

Types of matter	Equations of motion	Effect of $R^{\text{intrinsic}}$	Use of $R^{\text{intrinsic}}$
Timelike (Ordinary Matter)	Eq. (4) Einstein Field equations	Already contributes to the coefficient $\frac{8\pi G}{c^4}$	Renormalization of R[10]
Spacelike (Black Holes)	No equations of motion are defined	Only this scalar curvature present	$R^{\text{intrinsic}} = -3/r_0^2$

We will show that Eq. (5) is responsible for the internal pressure supporting BH, due to creating a constant negative scalar curvature inside a BH and cancels out the self-gravity inward pressure. Furthermore, the BH equilibrium is stable.

4. Black Hole Pressures

Because the BH is spacelike matter, there is no equation of static equilibrium and no equations of state. In order to discuss BH pressures then, we only have gravitational invariants at hand.

Because the Einstein field equations (4) remain intact for timelike matter, all of the experimental General Relativity experiments are maintained by the nonlinear joint action equation (3). The new equation (5) gives rise to a BH gravitational invariant intrinsic negative scalar curvature (Fig. 2)[6]

$$R = -3/r_0^2. \tag{8}$$

We take a moment and describe how Eqs. (4) and (5) work for a BH. Inside the BH, only Eq. (5) exists because it is a gravitational invariant. Outside the BH, $R = 0$, and Eq. (5) does not contribute. Now Eq. (4) has non-trivial $R = 0$ fields, which is the Schwarzschild solution.

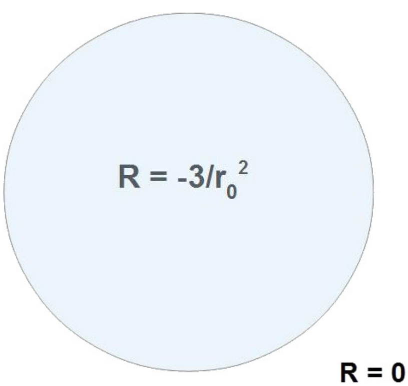

Fig. 2. A static BH has a constant negative scalar curvature inside the Horizon, and zero curvature outside. The scalar curvature is discontinuous at the Horizon. This scalar curvature is the origin of the pressure keeping a BH inflated.

It is the existence of this negative scalar curvature that gives rise to the following properties discussed below:

(1) Identifies a new BH physical constant that inflates the BH.
(2) Creates an outward pressure canceling out the BH inward self-gravity pressure.
(3) Explains why BH do not have a maximum mass.
(4) Allows the computation of the largest pressure in the Universe.

In order to derive the matter pressure for a Schwarzschild BH, we add to the first law of Black Hole Mechanics (conservation of energy)[6,11] the enthalpy work term VdP_M[11]

$$c^2 dM = VdP_M \tag{9}$$

which gives the change in mass (ordinarily it would be the change in energy dE) of the Black Hole due to the change in matter pressure P_M. Using Eq. (8),

$$dM = \frac{\alpha M^3}{6} dR, \quad \alpha = \frac{4G^2}{c^4}. \tag{10}$$

From Eq. (9)

$$\frac{c^2 \alpha M^3}{6} dR = VdP_M, \quad V = \frac{4\pi}{3} r_0^3. \tag{11}$$

The gravitational invariant (observable) that P_M is related to, is the Black Hole scalar curvature R

$$P_M = \beta R. \tag{12}$$

From Eq. (11),

$$\frac{dP_M}{dR} = \beta = \frac{c^2 \alpha M^3}{6V} \tag{13}$$

giving

$$P_M = -\frac{3}{8\pi} \frac{Mc^2}{r_0^3} = -\frac{3c^8}{64\pi G^3 M^2}. \tag{14}$$

Equation (14) is the negative pressure (enthalpy term) associated with collapsing spacelike matter of a static Schwarzschild Black Hole. The solution shows that the matter (enthalpy) pressure inside a static BH is a constant negative pressure and a gravitational invariant.

We give a derivation of the outward pressure P_S due to the negative scalar curvature. Both quantities are gravitational invariants. The pressure being exerted by scalar curvature R of Eq. (8), for any $g_{\mu\nu}$, has to satisfy the following equation, because of tensor equalities

$$\begin{aligned} -\frac{R}{2} g_{\mu\nu} &= \frac{8\pi G}{c^4} g_{\mu\nu} P_s, \\ P_S &= \frac{3}{2r_0^2} \frac{c^4}{8\pi G}, \end{aligned} \tag{15}$$

$$P_S = (\text{equilibrium equation (1)}) = \frac{3}{64\pi}\frac{c^8}{G^3 M^2}. \tag{16}$$

Thus we find that the total pressure inside a BH is zero

$$P_S = -P_M. \tag{17}$$

Equation (17) represents pressure equilibrium. From Table 1, we see now that the original Einstein–Hilbert action equation (2), which only gives Eq. (4), fails to explain why BH do not collapse to a mathematical singularity, since $R^{\text{intrinsic}}$ is missing.

We must show that the BH equilibrium is stable. If we make a small negative radius perturbation $r = r_0 - \zeta$, $\zeta > 0$, the volume changes dV, however the mass pressure equation (14) for P_M does not change, because the work with constant P_M, $P_M dV$, goes into a smaller mass. However, now, from Eq. (16) $|P_S| > |P_M|$ because P_S has an added pressure from Eq. (1). The two pressures are no longer in equilibrium.

The balance equation between the attractive self-energy and the outward pressure with ρ the mass density

$$\rho \cdot \text{grad}(\Phi) = \text{grad}(P_S) \tag{18}$$

is the surface of constant gravitational potential Φ. This gives

$$\begin{aligned} &P_S = \text{constant},\ \text{Eq. (16)} \\ &\Phi \Rightarrow \text{constant} \end{aligned} \tag{19}$$

and a constant gravitational potential inside.

It is instructive now to list the differences between BH and NS, see Table 2.

In Table 3, we list the values for a static BH in equilibrium.

Table 2. Significant differences between Neutron Stars and Black Holes.

Black hole	Neutron star
Interior matter is spacelike	Interior matter is timelike
Constant outward pressure preventing collapse	monotonic pressure preventing collapse
Interior matter has no causality	Interior matter is causal
No temperature[6]	NS cools with time
BH has a negative scalar curvature[6]	

Table 3. Static BH equilibrium parameters (all gravitational invariants).

Property	Value
Energy density	$3c^8/32\pi G^3 M^2$
Scalar curvature R (from[6])	$-3c^4/4G^2 M^2$
Interior matter pressure P_M (First Law Black Hole Mechanics)	$-3c^8/64\pi G^3 M^2$
Interior spacetime pressure P_S Eq. (16)	$3c^8/64\pi G^3 M^2$

5. Contradictions Using Causal Physics with BH

BH has spacelike matter which is acausal. If you attempt to apply causal physics to it, you obtain contradictions. The Einstein equations of motion are causal and not valid for BH. If you nonetheless apply them, you obtain fake[12] singularities ($G = c = 1$ units)

$$R \overset{?}{=} -\frac{2M}{r^3}, \tag{20}$$

where R is the scalar curvature and r is the radial coordinate. Equation (20) is impossible because R is a gravitational invariant and coordinates are not.

Another contradiction arises if you apply finite temperature field theory (which is causal) to BH. Hawking made this mistake.[13] He concluded that BH have a temperature inversely proportional to its mass and therefore suffered thermal radiation, leading to a totally evaporated BH. Going back to Fig. 2, if the BH mass evaporates to 0, the negative scalar curvature $R \to -\infty$, which is not the vacuum state. This is the contradiction.

6. Black Hole Universal Constant

We find that a BH intrinsic negative curvature gives rise to a spacetime pressure that balances the BH self-gravity. It is a restoring pressure under radial perturbations and has solution equation (16)

$$P_S = \frac{3c^4}{4G} \times \frac{1}{4\pi r_0^2}, \tag{21}$$

$$P_S = \mathcal{F}/\text{area}, \tag{22}$$

where the universal constant $\mathcal{F}$ has the value

$$\mathcal{F} = \frac{3c^4}{4G} = 9.077\ldots \times 10^{43}\ \text{N} \tag{23}$$

and area is the area of the Horizon. The existence of the universal constant equation (23) has the following ramifications.

(2) Since $\mathcal{F}$ is independent of mass, there is no maximum mass for BH. As BH mass becomes large, its interior pressure decreases, all the while in pressure equilibrium.

(2) The highest pressure P_{Universe} in the Universe is a calculable physical quantity and is the pressure associated with the smallest BH. From the previous determination of the most massive static NS of $2.34\,\text{M}_\odot$, the smallest BH is $2.35\,\text{M}_\odot$. This gives

$$P_{\text{Universe}} = 1.5183\ldots \times 10^{35}\,\text{N/m}^2. \tag{24}$$

To get an idea of how large this pressure is, we compare it to the pressure thought to exist at the center of Jupiter[14] P_{Jupiter}

$$P_{\text{Jupiter}} = 650 \times 10^6 \text{ pounds/in}^2. \tag{25}$$

This gives

$$\frac{P_{\text{Universe}}}{P_{\text{Jupiter}}} = 3.3878 \times 10^{22}. \tag{26}$$

7. Coalescing Black Holes

Finally, we want to show how the existence of $\mathcal{F}$ controls the coalescence of BH. Starting off with two BH coalescing, the pressure in the coalescing volume V before fusion is $P_1 + P_2$. After coalescence, the pressure in the volume is P_3. In order to form a remanent with pressure P_3 (remembering that small BH have higher pressure)

$$P_1 + P_2 > P_3. \tag{27}$$

This determines the maximum releasable energy. With constant $\mathcal{F}$, Eq. (27) becomes the equivalent equations

$$\frac{1}{A_1} + \frac{1}{A_2} > \frac{1}{A_3}, \tag{28}$$

$$\frac{1}{M_1^2} + \frac{1}{M_2^2} > \frac{1}{M_3^2}. \tag{29}$$

In the unlikely event that $N > 2$ BH coalesce, this equation becomes

$$\frac{1}{A_1} + \frac{1}{A_2} + \cdots > \frac{1}{A_3}. \tag{30}$$

Equation (27) is an exothermic reaction and so gravitational waves carry off the excess energy.

In order to understand the physical process taking place, it is advantageous to put the pressure constraint as a pressure ratio $(P_1 + P_2)/P_3 > 1$ and view the *time-reversal* of this coalescence (its backward process), where M_3 splits into $M_2 + M_1$ with now gravitational wave energy flowing into the backward process. The pressure ratio, which is showing that higher pressure is required for the backward reaction, and the now gravitational wave energy flowing in, reveals that the backward reaction is the familiar gas turbine compressing gas. Thus the physical meaning of the forward process, the original coalescence, is that it is an inverse gas turbine. The experimental data[15] on the GW150914 BH merger is consistent with Eq. (29). It is worth pointing out that this discussion involves no entropy.

8. Conclusion

BH are spacelike matter and acausal. If you apply causal physics to spacetime matter, you get contradictions: fake singularities with the causal equations of motion and fake BH evaporation with the causal finite temperature field theory. We conclude with the following items:

(1) The scalar curvature R is invariant under isometric symmetries (distance invariance) associated with metric spaces. Gravitational Riemannian manifolds are metric spaces. The Standard Model L_{SM} has the same isometric symmetry and the joint action predicts that $R^{\mathrm{intrinsic}}$ and L_{SM} are proportional to each other, with the constant of proportionality $\frac{8\pi G}{c^4}$.
(2) In finite temperature field theory, the thermodynamic potential Ω and the gravitational scalar R have the same renormalization theorem for timelike matter: Both are physical observables without any Green's function external lines (fields).
(3) BH have a universal constant $\mathcal{F}$, independent of their mass. The smallest mass BH has the largest pressure in the Universe.
(4) Timelike matter has an upper bound to its mass, because of causality. Spacelike matter has no upper bound to its mass, because of the existence of $\mathcal{F}$.
(5) Because of causality, the planet Jupiter is a much more complex object than Black Holes. Because Jupiter is timelike matter and causal, the interior model requires the equation of state with the equation of hydrostatic equilibrium. Jupiter has an onion structure of different equations of state. This gas planet is assumed to have a heavy-element core surrounded by a hydrogen-helium envelope.[16,17] The envelope is further divided into metallic hydrogen inner envelope and its molecular form outer envelope. Different equations of state give you different distributions of heavy elements and helium. These in turn give you different gravitational moments. Contrast this with a Black Hole's constant mass density of undeterminable composition, with no equation of state, no equation of hydrostatic equilibrium and no phase transitions present.

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